Partie B

$$\begin{cases} f(x) = \dfrac{x}{x+\ln x} & \text{si } x>0 \text{ et } x\neq \alpha \\ f(0) = 0 \end{cases}$$

1) * $\lim\limits_{x\to 0^+} f(x) = \lim\limits_{x\to 0^+} \dfrac{x}{x+\ln x} = 0 = f(0)$

d'ou f est continue à droite en 0.

* $\lim\limits_{x\to 0^+} \dfrac{f(x)-f(0)}{x-0} = \lim\limits_{x\to 0^+} \dfrac{1}{x+\ln x} = 0$

d'ou f est dérivable à droite en 0 et $f'_d(0) = 0$.

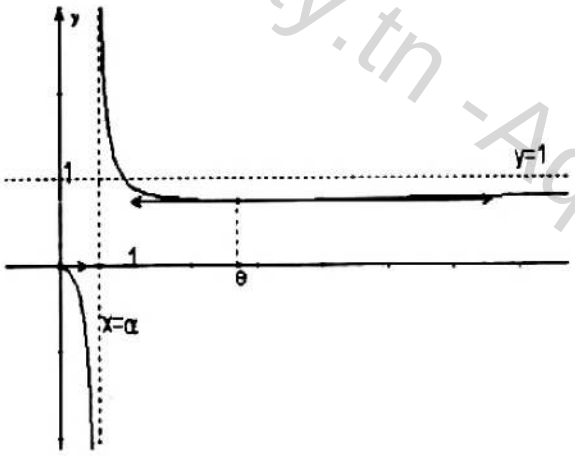
2) $f'(x) = \dfrac{(x+\ln x) - x(1+\frac{1}{x})}{(x+\ln x)^2} = \dfrac{\ln x - 1}{(x+\ln x)^2}$

$$\lim_{x\to+\infty} f(x) = \lim_{x\to+\infty} \frac{1}{1+\frac{\ln x}{x}} = \frac{1}{1+0} = 1 = 1$$

$$\lim_{x\to\alpha^+} f(x) = \lim_{x\to\alpha^+} \frac{x}{x+\ln x} = \frac{\alpha}{0^+} = +\infty$$

$$\lim_{x\to\alpha^-} f(x) = \lim_{x\to\alpha^-} \frac{x}{x+\ln x} = \frac{\alpha}{0^-} = -\infty$$

x	$-\infty$	α	e	$+\infty$
f'(x)	−	‖ −	0 +	
f(x)	0 ↘	$-\infty$ | $+\infty$ ↘	$\frac{e}{e+1}$ ↗	1



Partie C

$n \in IN^*$, $U_n = \int_1^e (\frac{\ln x}{x})^n dx$

1) $U_1 = \int_1^e \frac{1}{x}.\ln x\,dx = \left[\frac{1}{2}(\ln x)^2\right]_1^2 = \frac{1}{2}(\ln 2)^2$

2) $U_2 = \int_1^e \frac{1}{x^2}.(\ln x)^2 dx =$

$$\begin{cases} u'(x) = \dfrac{1}{x^2} \longrightarrow u(x) = -\dfrac{1}{x} \\ v(x) = (\ln(x))^2 \longrightarrow v'(x) = \dfrac{2}{x}\ln x \end{cases}$$

$$U_2 = \left[-\frac{1}{x}(\ln x)^2\right]_1^2 + \int_1^2 \frac{2}{x^2}.(\ln x).dx$$

$$= -\frac{1}{2}(\ln 2)^2 + \int_1^2 \frac{2}{x^2}.(\ln x).dx$$

$$\begin{cases} u'(x) = \dfrac{2}{x^2} \longrightarrow u(x) = -\dfrac{2}{x} \\ v(x) = \ln x \longrightarrow v'(x) = \dfrac{1}{x} \text{ d'où} \end{cases}$$

$$U_2 = -\frac{1}{2}(\ln 2)^2 + \left[-\frac{2}{x}\ln x\right]_1^2 - \int_1^e (-\frac{2}{x^2}).dx$$

$$U_2 = -\frac{1}{2}(\ln 2)^2 - \ln 2 - \left[\frac{2}{x}\right]_1^2$$

$$\boxed{U_2 = 1 - \ln 2 - \frac{1}{2}(\ln(2))^2}$$

3) $S_n(x) = 1 - \frac{\ln x}{x} + (\frac{\ln x}{x})^2 + \ldots.+(-1)^n(\frac{\ln x}{x})^n$

Somme des (n + 1) termes d'une suite géométrique de raison $(-\frac{\ln x}{x})$

$$S_n(x) = \frac{1-\left(-\frac{\ln x}{x}\right)^{n+1}}{1+\frac{\ln x}{x}} = \frac{x}{x+\ln x}\left[1-(-1)^{n+1}.\left(\frac{\ln x}{x}\right)^{n+1}\right]$$

4) $I_n = 1 - U_1 + U_2 - \ldots.. + (-1)^n U_n$

$$I = \int f(x).dx$$

a) $S_n(x) = f(x)\left[1-(1)^{n+1}(\frac{\ln x}{x})^{n+1}\right]$

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